

Exercise 23Find y' and y'' .

$$y = \sqrt{x} \ln x$$

Solution

Take the derivative of the function.

$$\begin{aligned} y' &= \frac{d}{dx} (\sqrt{x} \ln x) \\ &= \left[\frac{d}{dx} (\sqrt{x}) \right] \ln x + \sqrt{x} \left[\frac{d}{dx} (\ln x) \right] \\ &= \left(\frac{1}{2} x^{-1/2} \right) \ln x + \sqrt{x} \left(\frac{1}{x} \right) \\ &= \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}} \\ &= \frac{\ln x + 2}{2\sqrt{x}} \end{aligned}$$

Take another derivative.

$$\begin{aligned} y'' &= \frac{d}{dx} (y') \\ &= \frac{d}{dx} \left(\frac{\ln x + 2}{2\sqrt{x}} \right) \\ &= \frac{\left[\frac{d}{dx} (\ln x + 2) \right] (2\sqrt{x}) - \left[\frac{d}{dx} (2\sqrt{x}) \right] (\ln x + 2)}{(2\sqrt{x})^2} \\ &= \frac{\left(\frac{1}{x} \right) (2\sqrt{x}) - \left(2 \cdot \frac{1}{2} x^{-1/2} \right) (\ln x + 2)}{4x} \\ &= \frac{2x^{-1/2} - x^{-1/2} \ln x - 2x^{-1/2}}{4x} \\ &= \frac{-x^{-1/2} \ln x}{4x} \\ &= -\frac{\ln x}{4x^{3/2}} \end{aligned}$$